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Instrument for the Continuous Measurement of the Density of Flowing Fluids*

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This paper describes the development of a new electromechanical method of measuring the densities of flowing cryogenic fluids. The instrument uses a movable section of flow passage, vibrated transversely at a constant amplitude and frequency, as the sensing element. A dynamometer, inserted between the flow passage and driver, continuously measures the acceleration reaction (product of mass and acceleration) of the fluid in the passage. A measure of this reaction is also a measure of the fluid density in the passage. Performance results indicate that the densitometer should be well suited for service in liquid oxygen and nitrogen single- and two-phase flow systems, and with only minor changes for use with liquid hydrogen.

I. INTRODUCTION

DURING recent years, a lively interest in mass fluid flow measurement has been created. This is due, in part, to the desirability of knowing the mass flow rates of cryogenic fuels and oxidizers to rocket engines during tests. The changes in density which can occur during transfer of the fluids make volume flow measurement unsatisfactory.

Frequently mass flow rate is determined by obtaining a product of density and volume flow rates (inferential method).[1] The major difficulty with this method has been the lack of a satisfactory technique for measuring the density of a flowing fluid.[2] The purpose of the work reported here is to develop a simple, reliable instrument capable of providing continuous information on the density of a flowing cryogenic fluid.

The principal advantages of the instrument developed are its simplicity, good frequency response, and ability to measure densities of either single- or two-phase fluids. In addition, measurement is made on actual flow, not on a sample whose accuracy is doubtful.

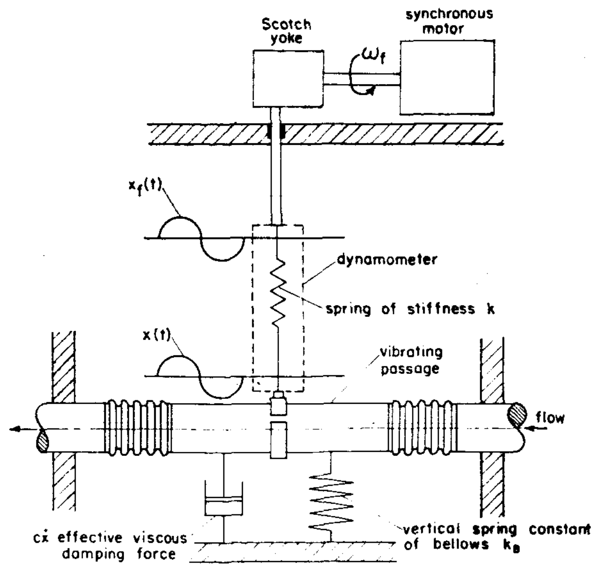

FIG. 1. Dynamic equivalent of densitometer.

II. THEORY OF OPERATION

The underlying idea of the densitometer is that the mass of any vibrating system is a primary factor in determining the dynamic characteristics of the system. If the system is designed so that the fluid flowing through it measurably affects the vibrating mass, a means of measuring fluid densities will have been provided.

Figure 1 illustrates schematically the system employed. The passage is supported by flexible bellows having a transverse spring constant k_B, and is driven with a sinusoidal motion ($x_0 \sin\omega_f t$) via a spring of stiffness k. The vertical motion of the passage is $x(t)$. The effective damping force, assumed to be viscous, is designated by $c\dot{x}(t)$. Assuming the fluid in the passage behaves as a rigid body, the

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[1] C. C. Miesse, "Study of Mass Flowmeters" Armour Research Foundation of Illinois Institute of Technology Project D173 (June, 1959).

[2] L. R. Favero *et al.*, "Hydrogen Mass Flowmeter Development," Aerojet-General Corporation Interim Progress Report 0356-01-1 (March, 1960).

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acceleration reaction of the mass is $M\ddot{x}(t)$, where M is the total vibrating mass of the system ($M=m+\rho V$) and m is the tare mass. V is the volume of the fluid affected by the motion of the passage, and ρ the density of the fluid. Writing the equation of motion we get

$$M\ddot{x}(t)+C\dot{x}(t)+(k+k_B)x(t)=kx_0\sin\omega_f t. \tag{1}$$

This is the familiar classical equation for a vibrating system having one degree of freedom. The formal steps of its solution may be found in many standard reference works[3] and will not be included here. One form of the particular solution readily obtained is

$$x(t)=\frac{x_0\cos\alpha}{[1+k_B/k-M\omega_f^2/k]}\sin(\omega_f t-\alpha), \tag{2}$$

where the angle α is the phase angle between the motion of the passage and driver, and is governed by the following equation:

$$\alpha=\tan^{-1}\left\{2\left(\frac{\omega_f}{\omega_n}\right)\left(\frac{c}{c_c}\right)\Big/1-\left(\frac{\omega_f}{\omega_n}\right)^2\right\}. \tag{3}$$

In order that the dynamometer output be a linear function of density it is necessary that α be negligibly small. By employing a stiff dynamometer ($k\gg M\omega_f^2$), frequency ratios (ω_f/ω_n) on the order of 1/2000 can readily be obtained. This is sufficient for a subcritically damped system ($c/c_0<1$) to cause an inphase motion between the driver and passage. Assuming the terms mentioned are negligible, Eq. (2) reduces to

$$x(t)=\left[\frac{x_0}{1+(k_B/k)}\right]\sin\omega_f t. \tag{4}$$

It is noted that the amplitude of the vibrated passage will differ from that of the driver by an amount determined from the ratio k_B/k. Since the sensitivity of the instrument is directly proportional to the amplitude of the passage, it is desirable that the amplitude be maximum; therefore, a design criterion is $k\gg k_B$.

Solving for the maximum force exerted on the dynamometer gives

$$F_{\max}=k_Bx_0-x_0M\omega_f^2. \tag{5}$$

Solving Eq. (5) explicitly for the density, we get

$$\rho=a+bF_{\max}, \tag{6}$$

where a and b are constants defined by

$$a=(k_B-m\omega_f^2)/(V\omega_f^2),\quad b=-1/(x_0V\omega_f^2).$$

The dynamometer, sensing the force exerted by the passage, generates an electrical signal (E_{ac}) in phase with the motion and proportional to the maximum force, or

$$E_{ac}\sim F_{\max}\sin\omega_f t.$$

[3] S. Timoshenko, *Vibration Problems in Engineering* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1959).

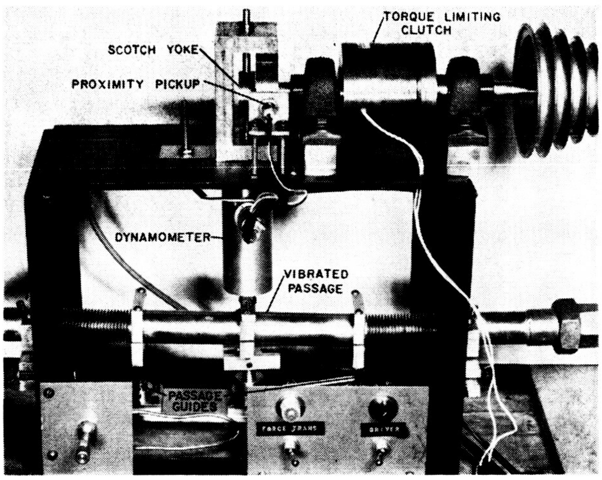

FIG. 2. Detailed view of first model.

If E_{ac} is rectified, the resulting signal (E_{dc}) is proportional to the maximum force, or

$$E_{dc}\sim F_{\max}.$$

The final relation for the fluid density takes the form

$$\rho=a+b'E_{dc}, \tag{7}$$

where a and b' are constants that can be determined experimentally.

III. DESCRIPTION OF DENSITOMETER

To investigate the practicality of the method outlined, a simple laboratory model was built and tested. A detailed view of this first model is presented in Fig. 2. Once sufficient information regarding its principle of operation was obtained, the cryogenic model was designed and built.

Details of the prototype are shown in the accompanying sectional drawing and photograph, Figs. 3 and 4. The vibrated fluid passage is approximately 9 in. in length, including both couplings, and has an effective volume of 7 cu in. The unit has an over-all length of 20 in., is 18 in. in height, and has an approximate weight of 50 lb (excluding readout instrumentation).

The motion of the passage is generated by a mechanical oscillator (Scotch yoke mechanism)[4] located directly above the dynamometer. The driving mechanism produces a sinusoidal motion having an amplitude of 0.1 in. and a frequency of 12 cps. Power to the mechanism is supplied by a 1/20-hp synchronous motor.

Single-ply beryllium copper bellows provide the necessary flexibility for the oscillated section of flow passage. The outboard ends of these bellows are fixed rigidly to the

[4] The mechanism is essentially a crank with a slotted cross head at right angles to the direction of rectilinear motion. If the crank rotates uniformly, the cross head will be given a harmonic motion. For details see, H. A. Rothbart, *Cams* (John Wiley & Sons, Inc., New York, 1956).

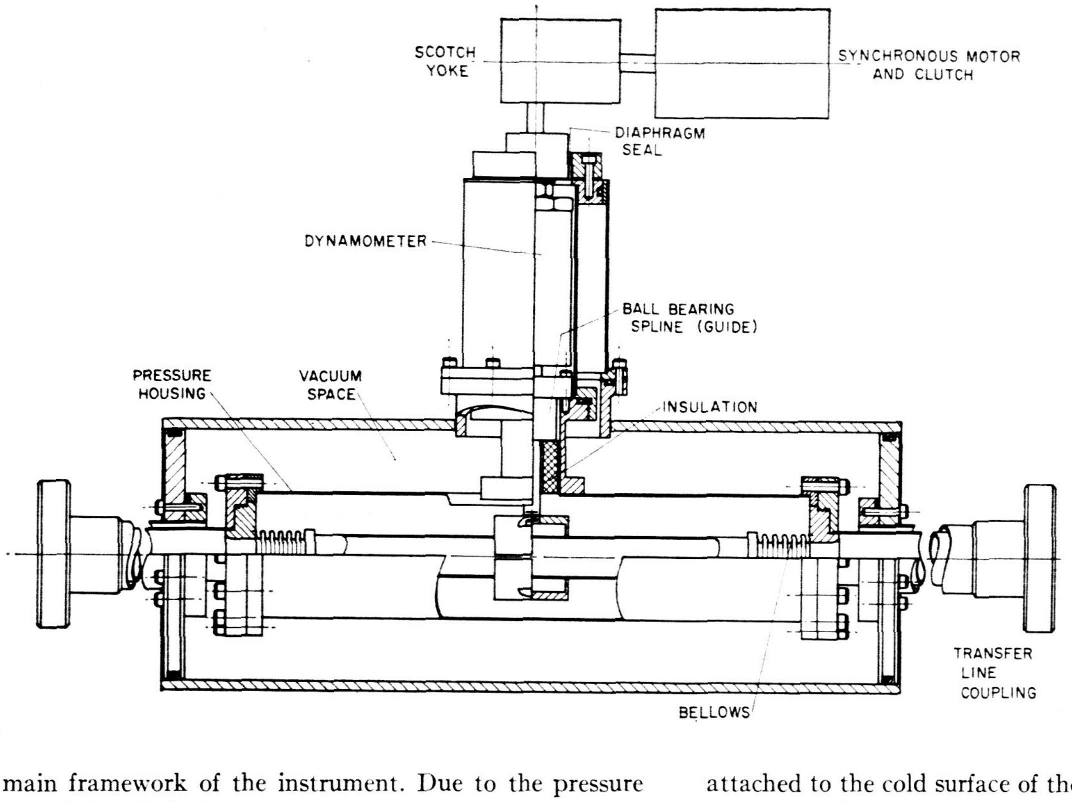

FIG. 3. Cross-section view of prototype.

main framework of the instrument. Due to the pressure sensitivity of the bellows the internal flow passage is connected, through a pressure equalizing tube, to the interior of the pressure housing. This prevents the existence of a pressure differential across the bellow walls, and thus eliminates any pressure sensitivity. A vacuum jacket completely surrounds this housing.

The dynamometer is mounted in the vertical plane between the driver and the flow tube. The gauge, which operates on the unbonded strain gauge principle, is 1.50 in. in diameter, 4 in. in length, and has a weight of approximately 1 lb. To minimize heat conduction, the gauge is attached to the cold surface of the flow passage by a small diameter, 3-in.-long stainless steel rod. A cylinder of Styrofoam is installed below the gauge to reduce convection currents in the tube surrounding the dynamometer. To protect the dynamometer in case of any malfunction, an overload clutch was incorporated into the drive assembly. This clutch is mounted between the oscillator and motor. When the output from the dynamometer exceeds a predetermined level, the clutch de-energizes, which stops the oscillator.

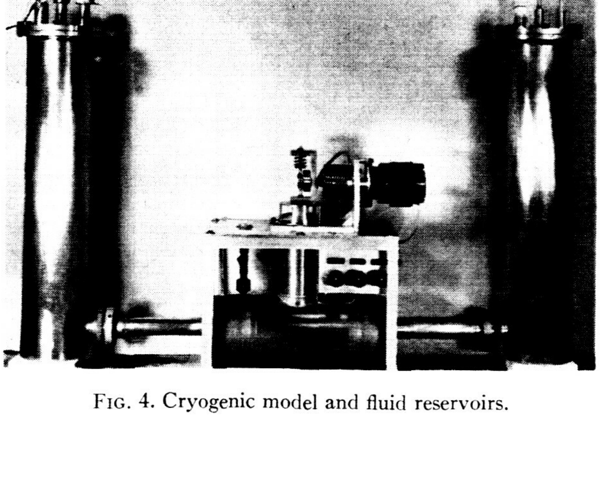

FIG. 4. Cryogenic model and fluid reservoirs.

Above the dynamometer is a flexible diaphragm and piston arrangement which serves as the seal for the pressure housing. The diaphragm is made of a rubber impregnated fabric capable of withstanding high pressures.[5]

A ball bearing spline assembly mounted immediately below the dynamometer is used to restrict the motion of the flow passage and dynamometer to one degree of freedom.

The electronic circuity is shown in Fig. 5. Power to the bridge is supplied by a 10-V constant voltage power supply. The 12-cps millivolt level signal from the force gauge is first filtered by a filter designed to pass frequencies ranging from 11–13 cps. The signal is then amplified by a factor of several hundred, rectified, and refiltered to remove ripple. Either an analog or digital voltmeter can be used as the final readout equipment. The particular one chosen

[5] The diaphragm, manufactured by the Bellofram Corporation, is a nonporous elastomeric membrane which provides a frictionless rolling pressure seal between the piston and side wall. Seals are available which have working pressures up to 1200 psi.

would depend on the specific application and the degree of accuracy desired.

IV. EVALUATION AND DISCUSSION

Because both single-phase and two-phase flows exist in cryogenic systems, the densitometer was evaluated for both types of fluids. Preliminary measurements under non-flow conditions were made by filling the test passage with ambient temperature fluids using the fluid reservoirs shown in Fig. 4. Five single-phase fluids were used in the initial calibration (see caption Fig. 6). Liquids were selected to give a range of specific gravity from 0.65 to 1.30.

The two-phase test employed air and water, and consisted of draining measured quantities of water from an initially filled passage. Knowing the total mass of water remaining and the total volume occupied by the air–water mixture, the average or bulk density was calculated. Figure 6 shows the results of the ambient temperature calibration obtained with the single- and two-phase fluids.

Static evaluation of the prototype at 76°K (liquid-nitrogen temperature) was accomplished by using weights, cooled by nitrogen gas, instead of liquid nitrogen. To best understand this method, refer to the governing equation

$$\rho=(k_B-m\omega_f^2)/(V\omega_f^2)-F(t)_{\max}/(x_0V\omega_f^2). \tag{6}$$

From the above equation we see that by maintaining a constant negligible density fluid in the sensing tube (which can be realized by using a low density gas to cool the flow passage), the zero shift as a function of temperature can be readily determined. Any number of additional effective densities can be obtained by simply varying the tare mass through the addition of weights to the vibrating passage. For example, if an additional mass m_0 is added to mass m, Eq. (6) may be rewritten as

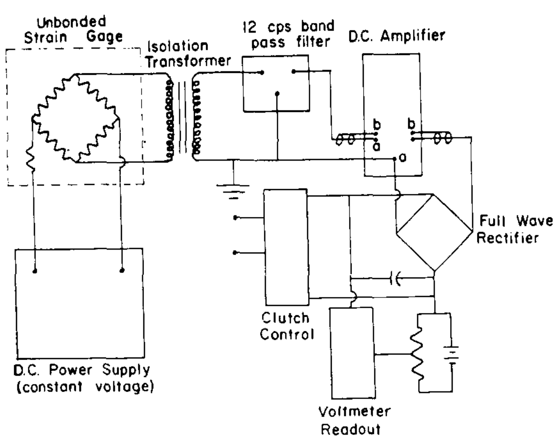

FIG. 5. Electronic circuitry.

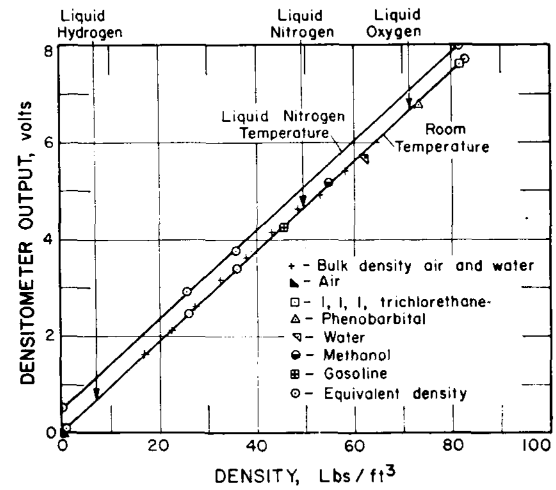

FIG. 6. Densitometer calibration curve.

$$\rho_0\equiv\rho+\frac{m_0}{V}=\frac{k_B-(m+m_0)\omega_f^2}{V\omega_f^2}-\frac{F(t)_{\max}}{x_0V\omega_f^2}, \tag{6'}$$

where ρ_0 is defined as the equivalent density.

Figure 6 contains the plot of equivalent density versus densitometer output obtained from tests performed at room and liquid-nitrogen temperatures. The liquid-nitrogen temperature curve shows a zero shift of approximately 5% full scale from the room-temperature curve. This change is of the same magnitude as predicted from considering the change in the elasticity of the bellows between room and liquid-nitrogen temperatures. No change in sensitivity was observed. Cool-down caused no measureable change in the mechanical performance of the instrument. In addition, thermal cycling between room temperature and 76°K had no apparent effect on the densitometer performance.

Since the densitometer has been evaluated over a wide range of temperatures and density, it is the authors' opinion that the instrument is well suited for service in liquid oxygen and nitrogen systems. Application of the densitometer in liquid-hydrogen systems would require replacing the present dynamometer with one having a larger sensitivity. Performance results from the static calibration tests indicated an instrument accuracy of better than ±2% full scale. However, further refinements in the electronic circuity and possibly the driving mechanism should cause a substantial improvement over the present accuracy.

Actual flow tests using water were conducted to examine the behavior of the densitometer under actual flow conditions. Flow velocities up to 7 fps were established in the sensing tube. Prior to measurements, care was taken to see that no air pocket remained in the flow passage. Test results showed the densitometer is insensitive to flow and that drag and momentum effects were not detected.